Electromagnetic Waves

Maxwell's eqns $\Rightarrow$ $$\nabla^2 \vec{E} = \frac{1}{v^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$
$$\nabla^2 \vec{B} = \frac{1}{v^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \ m/s \equiv c$$

and: $\vec{E} \perp \vec{B}$, $\frac{|E|}{|B|} = c$ at each point in space

$\vec{E} \times \vec{B}$ points in direc. of propogation

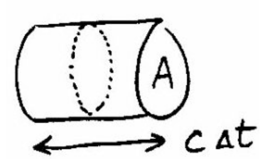
Poynting vector:

energy density in field: $u_E = \frac{\varepsilon_0}{2} E^2$ $\left(J/m^3\right)$

$$u_B = \frac{1}{2\mu_0} B^2 = u_E$$

$$u_{tot} = u_E + u_B = \varepsilon_0 E^2 = B^2/\mu_0$$

energy flux:



$$\frac{uAc\Delta t}{A\Delta t} = uc \equiv S \quad \left(\frac{W}{m^2}\right)$$

or $S = \varepsilon_0 E^2 c = \varepsilon_0 E B c^2 = \dfrac{EB}{\mu_0}$

assume $\vec{S} \parallel \vec{k}$ (isotropic media)

$$\Rightarrow \boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}} = c^2\varepsilon_0\,\vec{E}\times\vec{B}$$

eg. plane (harmonic) wave,

$$\vec{E} = \vec{E}_0\cos(\vec{k}\cdot\vec{r}-\omega t)$$
$$\vec{B} = \vec{B}_0\cos(\vec{k}\cdot\vec{r}-\omega t)$$

$$\vec{S} = c^2\varepsilon_0\,\vec{E}_0\times\vec{B}_0\,\underbrace{\cos^2(\vec{k}\cdot\vec{r}-\omega t)}_{\nu\approx 10^{15}\,s^{-1}\ \text{rapidly varying!}}$$

time average:

$$\left\langle \cos^2(\vec{k}\cdot\vec{r}-\omega t)\right\rangle_T = \frac{1}{T}\int_{t-T/2}^{t+T/2}\cos^2(\vec{k}\cdot\vec{r}-\omega t)\,dt$$

$$=\frac{1}{T}\int_{t-T/2}^{t+T/2}\left(\frac{1}{2}+\frac{1}{2}\cos\left[2(\vec{k}\cdot\vec{r}-\omega t)\right]\right)dt$$

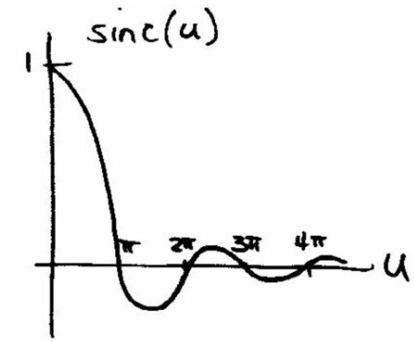
$$= \frac{1}{2}\left(\frac{T}{T}\right)-\frac{1}{2T\omega}\underbrace{\left\{\sin\left[2\left(\vec{k}\cdot\vec{r}-\omega t-\omega\tfrac{T}{2}\right)\right]-\sin\left[2\left(\vec{k}\cdot\vec{r}-\omega t+\omega\tfrac{T}{2}\right)\right]\right\}}$$

$$\sin(\alpha-\beta)-\sin(\alpha+\beta) = -\cos\alpha\sin\beta$$

$$= \frac{1}{2}+\frac{1}{2\omega T}\cos\left[2(\vec{k}\cdot\vec{r}-\omega t)\right]\sin(\omega T)$$

$$= \frac{1}{2}\left\{1+\frac{\sin(\omega T)}{\omega T}\cos\left[2(k\cdot r-\omega t)\right]\right\}$$

define as: $\mathrm{sinc}(\omega T)$



most measurement devices average over $T \gg \dfrac{1}{\omega}$

$$\Rightarrow \langle S\rangle_T = \frac{c^2\varepsilon_0}{2}\left|\vec{E}_0\times\vec{B}_0\right| = \frac{c\,\varepsilon_0 E_0^2}{2}$$

$\equiv I$, irradiance

Average energy per area per time.

for complex form of E,

$$E = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \quad I = \frac{\varepsilon_0 c}{2}\underbrace{EE^*}_{E_0^2}$$

(E_0 real)

no time av. needed!

optical power or "radiant flux" $\quad P = \int I\, dA \quad (W)$

$$\frac{\text{incident power}}{\text{area}} = \text{irradiance}$$

$$\frac{\text{exiting power}}{\text{area}} = \text{exitance}$$

both are flux densities (or intensities)

check: spherical source

$$E = \frac{A}{r}\cos(kr - \omega t)$$

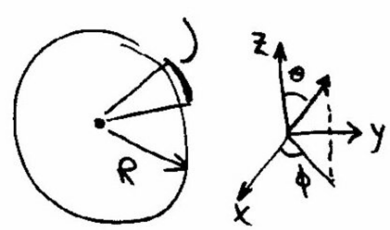
$$\langle E^2 \rangle_T = \frac{1}{2}\frac{A^2}{r^2}$$

$$I = \varepsilon_0 c \frac{1}{2}\frac{A^2}{r^2}$$

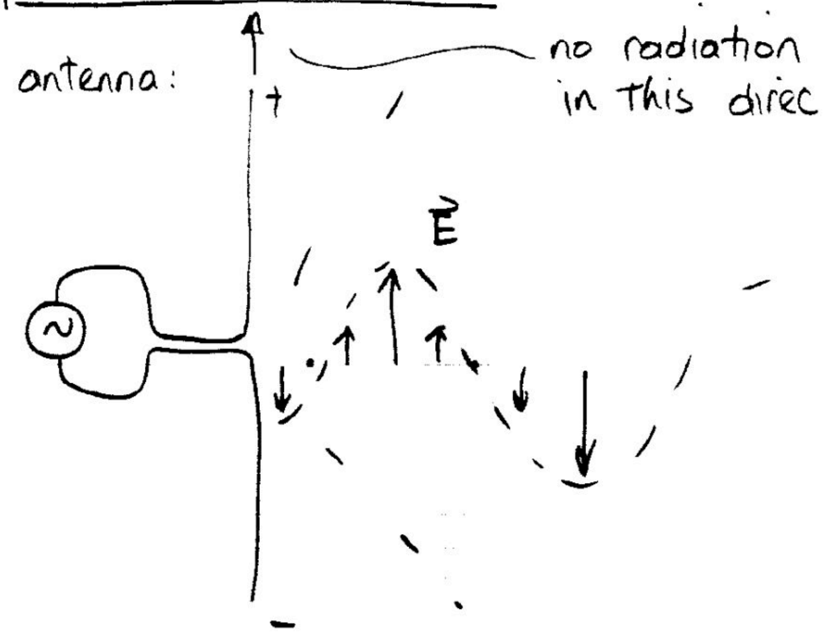
$$P = \int \cancel{R^2} \sin\theta\, d\phi\, d\theta\, \varepsilon_0 c \frac{1}{2}\frac{A^2}{\cancel{R^2}}$$

$$= 2\pi \varepsilon_0 c A^2 = \text{const.} \quad \checkmark$$

$dA = r^2 \sin\theta\, d\phi\, d\theta$



production of EM waves:



also, drive atom:

(add to total)

Photons:

- light comes in a minimum size bundle of energy, a "photon":

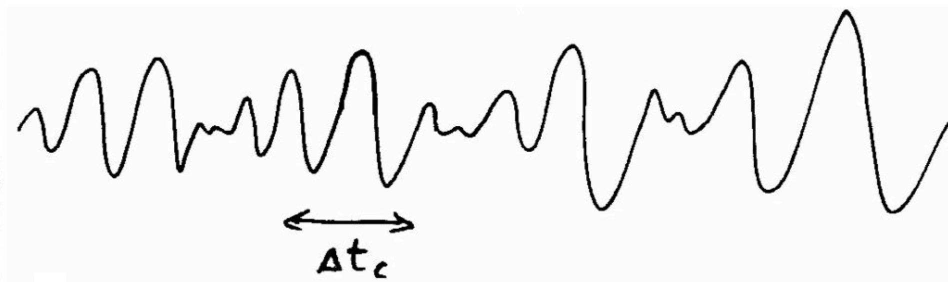
$$\mathcal{E} = h\nu \quad \left(\begin{matrix}\text{visible}\\ \text{light}\end{matrix},\ \approx 2\ \text{eV}\right)$$

$\Rightarrow$ as reduce intensity of a lamp, eventually will observe individual photons being emitted

(photon counting; easy with photomultiplier tube).

for fixed Δt_c there is a minimum E_0 possible!

train of photons from quasimonochromatic source (eg. discharge lamp):



(each "packet" may be one or several photons)

$\rightarrow$ phase changes randomly <u>and</u> direc. of $\vec{E}$ changes randomly.

<u>back to flux</u>:

photon flux $\Phi = \dfrac{P}{h\nu} = \dfrac{IA}{h\nu}$

eg. 1 W, 2 eV photons, $\Phi = \dfrac{1\ \text{J/s}}{2\cancel{\text{e}}\text{V} \times 1.6\times 10^{-19}\ \text{C}}$

$\approx 3 \times 10^{18}\ \dfrac{\text{photons}}{\text{s}}$